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Formulas at Your Fingertips

Asset Allocation

Capital Market Expectations, Part 1: Framework and Macro Considerations

- Aggregate Equity Market Value

$V_t^e = \text{GDP}_t \times S_t^k \times PE_t$

$= \text{GDP}_t \times \frac{E_t}{\text{GDP}_t} \times \frac{P_t}{E_t}$

Where GDP is gross domestic product, S^k equals capital's share of income (i.e, corporate earnings as a percentage of GDP), and PE is the price-to-earnings ratio.

- Taylor Rule

$i^* = r_{neutral} + \pi_e + 0.5(\hat{Y}_e - \hat{Y}_{trend}) + 0.5(\pi_e - \pi_{target})$

where:
i^* = target nominal policy rate
$r_{neutral}$ = real policy rate targeted with trend growth and target inflation
$\hat{Y}_e, \hat{Y}_{trend}$ = expected and trend real GDP growth rates
π_e, π_{target} = expected and target inflation rates

- Macroeconomic Linkages

$(X - M) = (S - I) + (T - G)$

where:
X = exports
M = imports
S = domestic saving
I = domestic investment
T = taxes
G = government spending

Capital Market Expectations, Part 2: Forecasting Asset Class Returns

- Grinold-Kroner Model

$E(R_e) = \frac{D}{P} + (\%\Delta E - \%\Delta S) + \%\Delta P/E$

where:
$\frac{D}{P}$ = Dividend yield
%ΔE = Nominal earnings growth rate
%ΔS = Expected percentage change in shares outstanding
%ΔP/E = Growth rate of the P/E ratio (the "repricing return")

Some points of note:

1. The term −%ΔS is referred to as the "rate of net share repurchases" and represents income from company buybacks. Hence, the "income" component of expected return is D/P − %ΔS.
2. Expected capital gains are composed of the nominal earnings growth rate, %ΔE, plus the repricing return, %ΔP/E.
3. The term (%ΔE − %ΔS) represents the estimated rate of change of earnings *per share*.

- Singer and Terhaar

Under full integration, every asset could be priced 100 percent against the global capitalization-weighted market portfolio:

$RP_i^G = \beta_{i,GM} RP_{GM} = \rho_{i,GM}\sigma_i\left(\frac{RP_{GM}}{\sigma_{GM}}\right)$

where:

$\beta_{i,GM} = \frac{\text{Cov}(R_i, R_{GM})}{\text{Var}(R_{GM})} = \rho_{i,GM}\left(\frac{\sigma_i}{\sigma_{GM}}\right)$

With less than perfect integration, the relationships are adjusted using the degree to which assets are integrated with the global portfolio:

$RP_i = \varphi RP_i^G + (1 - \varphi)RP_i^S$

where:
φ = degree of global integration
RP_i^G = risk premium under global equilibrium (integrated)
RP_i^S = risk premium under local-market equilibrium (segmented)

Considering the risk premium equals the product of asset standard deviation and the Sharpe ratio, the fully segmented, fully integrated, and partially integrated equations are:

$RP_i^G = \rho_{i,GM} \times \sigma_i \times \frac{RP_{GM}}{\sigma_{GM}}$

$RP_i^S = \sigma_i \times \textit{Sharpe Ratio}(i)$

$RP_i = \varphi RP_i^G + (1 - \varphi)RP_i^S$

- Capitalization Rate

The capitalization rate is like dividend yield for equities, and may be used in a similar way in developing expected return for the property:

$E(R_e) = \text{DIV YLD} + g_{DIV}$

$E(R_{re}) = \text{Cap rate} + g_{NOI}$

The long-term NOI growth rate, g_{NOI}, should be close to the nominal GDP growth rate. Short-term estimates may require an analyst to adjust the cap rate (rather than the NOI growth rate) to determine cyclical impacts:

$E(R_{re}) = \text{Cap rate} + g_{NOI} - \%\Delta\text{ Cap rate}$

Capitalization rates will tend to rise along with long-term interest rates and real estate values will therefore tend to be pro-cyclical.

- Capital Flows

Exchange rate differences will reflect differences in the short-term interest rates, term premiums, credit premiums, equity premiums, and liquidity premiums between two countries:

$E(\%\Delta S_{d/f}) = (r^d - r^f) + (\text{Term}^d - \text{Term}^f) + (\text{Credit}^d - \text{Credit}^f) + (\text{Equity}^d - \text{Equity}^f) + (\text{Liquidity}^d - \text{Liquidity}^f)$

Terms of the equation respectively relate to various parts of the diversified portfolio:

- Money market (nominal short-term rates)
- Government bonds (add term premium)
- Corporate bonds (add credit premium)
- Public equities (add equity premium)
- Private assets, including PP&E investment (add liquidity premium)

- Estimating Volatility from Smoothed Returns

$R_t = (1 - \lambda)r_t + \lambda R_{t-1}$

$0 < \lambda < 1$

$var(r) = \left(\frac{1+\lambda}{1-\lambda}\right)var(R)$

where:
R_t = observed return at time t
r_t = true return at time t

Overview of Asset Allocation

- Investor's Utility Function

$U_m = E(R_m) - 0.005\lambda\sigma_m^2$

where m is an asset allocation mix, U is the client's utility function, and λ is the client's coefficient of risk aversion.

- Portfolio Risk Budgeting

Marginal contribution to total risk identifies the rate at which risk changes as asset *i* is added to the portfolio:

$MCTR_i = \beta_{i,P}\sigma_P$

where:
$\beta_{i,P}$ = beta of asset *i* returns with respect to portfolio returns
σ_P = portfolio return volatility measure as standard deviation of asset *i* returns

Absolute contribution to total risk identifies the contribution to total risk for asset *i*

$ACTR_i = w_i \times MCTR_i$

$\%ACTR_i \text{ to total risk} = \frac{ACTR_i}{\sigma_P}$

The optimal portfolio occurs when:

$\frac{r_i - r_f}{MCTR_i} = \frac{r_j - r_f}{MCTR_j} = \dots = \frac{r_{TP} - r_f}{\sigma_{TP}}$

where:
σ_{TP} = standard deviation of the tangency portfolio

Asset Allocation with Real-World Constraints

- After-Tax Portfolio Optimization

Optimizing a portfolio subject to taxes requires using the after-tax returns and risks on an *ex-ante* basis.

$r_{at} = r_{pt}(1 - t)$

where:
r_{at} = expected after-tax return
r_{pt} = expected pre-tax return
t = expected tax rate

Extending this to a portfolio with both income and capital gains:

$r_{at} = p_d r_{pt}(1 - t_d) + p_a r_{pt}(1 - t_{cg})$

where:
p_d = proportion of return from dividend income
p_a = proportion of return from price appreciation (i.e., capital gain)
t_d = tax rate on dividend income
t_{cg} = tax rate on capital gain

This formula ignores the multi-period benefit from compounding capital gains rather than recognizing the annual capital gain.

Taxes also affect expected standard deviation.

$\sigma_{at} = \sigma_{pt}(1 - t)$

Taxes result in lower highs and higher lows, effectively reducing the mean return and muting dispersion.

- Equivalent After-Tax Rebalancing Range

$R_{at} = R_{pt}/(1 - t)$

where:
R_{at} = After-tax rebalancing range
R_{pt} = Pre-tax rebalancing range

- Portfolio Value After Taxable Distributions

$V_{at} = V_{pt}(1 - t_i)$

where:
v_{at} = after-tax portfolio value
v_{pt} = pre-tax portfolio value
t_i = tax rate on distributions as income

Portfolio Construction

Overview of Fixed-Income Portfolio Management

- Portfolio Measures of Risk

$$\text{Avg Mod Dur} = \sum_{j=1}^{J} w_j \text{Mod Dur}_j$$

where:
w_j = weight of bond *j* in the portfolio
Mod Dur_j = modified duration of bond *j*
J = number of bonds in the portfolio

$$\text{Avg Convexity} = \sum_{j=1}^{J} w_j \text{Convexity}_j$$

where:
Convexity_j = convexity of bond *j*

- Effective Duration and Convexity

$$\text{Effective Duration} = \frac{(PV_-) - (PV_+)}{2(\Delta\text{Curve})(PV_0)}$$

$$\text{Effective Convexity} = \frac{(PV_-) + (PV_+) - (2PV_0)}{(\Delta\text{Curve})^2 (PV_0)}$$

where:
PV_0 = current value of the portfolio
PV_- = value of portfolio when benchmark curve is shifted down
PV_+ = value of portfolio when benchmark curve is shifted up
ΔCurve = change in benchmark curve

- Components of Fixed Income Returns

Return can be decomposed into the sum of:

1. Coupon income: annual coupon / current bond price
2. Rolldown return, assuming no change in yield curve: [Projected ending bond price (B_1) – Beginning bond price (B_0)] / B_0
3. Price change due to benchmark yield changes: (–MD × ΔY) + (½C × ΔY^2)
4. Price change due to spread changes: (–MD × ΔS) + (½C × ΔS^2)
5. Currency G/L: change in value of foreign currencies weighted for exposure to the currency

Coupon income + rolldown return = rolling yield.

- Leveraged Portfolio Return

$$r_p = \frac{r_I(V_B + V_E) - (r_B \times V_B)}{V_E} = r_I + \frac{V_B}{V_E}(r_I - r_B)$$

where:
r_p = leveraged portfolio return
r_I = return on invested assets
V_B = value of borrowed funds
V_E = value of equity (i.e., investor's funds)
r_B = cost of borrowing (i.e., interest rate)

Asset Allocation to Alternative Investments

- Capital Contributions

$$C_t = RC_t \times (CC - PIC_t)$$

where:
C_t = Capital contribution
RC_t = Rate of contribution (%)
CC = Capital commitment
PIC_t = Paid-in capital

For example, 25% of the capital commitment (RC_1 = 25%) could be forecast for the first year and 50% of remaining commitment [50% × (CC – PIC_t)] could be forecast for subsequent years.

- Distributions

$$D_t = RD_t \times [NAV_{t-1} \times (1 + g)]$$

where:
$$NAV_t = [NAV_{t-1} \times (1 + g)] + C_t - D_t$$

The above formula assumes that NAV_{t-1} is calculated before adding contributions and subtracting distributions (i.e., contributions and distributions are not subject to growth for the calculation. The formula should be appropriately adjusted for contributions and distributions subject to the growth rate.

$$RD_t = (t/L)^B$$

where:
L = life of fund
B = bow parameter, which defines the shape of distributions over time

Portfolio Management for Institutional Investors

- Spending Rules for Endowments

$$\text{Spending amount}_{t+1} = w \times \text{Spending amount}_t \times (1 + \text{Inflation}) + (1 - w) \times \text{Spending rate} \times \text{Average AUM}$$

When *w* = 0 this becomes the market value rule.
When *w* = 1 this becomes the constant growth rule.

- Balance Sheet Management for Banks and Insurers

Percentage change in equity:

$$\frac{\Delta E}{E} = \frac{\Delta A}{A}\left(\frac{A}{E}\right) - \frac{\Delta L}{L}\left(\frac{A - E}{E}\right) = \frac{\Delta A}{A}\left(\frac{A}{E}\right) - \frac{\Delta L}{L}\left(\frac{A}{E} - 1\right)$$

where:
E = Equity
A = Assets
L = Liabilities

Duration of equity:

$$D_E^* = \left(\frac{A}{E}\right)D_A^* - \left(\frac{A}{E} - 1\right)D_L^*\left(\frac{\Delta i}{\Delta y}\right)$$

where:
D_E = duration of equity
D_A = duration of assets
D_L = duration of liabilities
($\Delta i/\Delta y$) = change in yield of liabilities relative to change in yield of liabilities

Variance of equity:

$$\sigma^2_{\frac{\Delta E}{E}} = \left(\frac{A}{E}\right)^2 \sigma^2_{\frac{\Delta A}{A}} + \left(\frac{A}{E} - 1\right)^2 \sigma^2_{\frac{\Delta L}{L}} - 2\left(\frac{A}{E}\right)\left(\frac{A}{E} - 1\right)\rho\sigma_{\frac{\Delta A}{A}}\sigma_{\frac{\Delta L}{L}}$$

where:
ρ = correlation of the changes in assets and liabilities